Table 1 Variables which may affect trace constituents

Total pressure	Allowable concentration of toxicants
Gaseous composition	Regenerative life support systems
Relative humidity	Emergency conditions
External radiation	Equipment stored aboard
Physical irritants	Launching, orbiting, and reentry forces
Foodstuffs	Background levels
Physical activity of crew	Detoxification mechanisms
Type and amount of absorbents aboard	Relative concentrations

Oxygen will continue to play a prominent role in man's atmosphere, and it can be assumed that oxidative processes will be occurring continually. Numerous air pollution studies have shown that incomplete oxidation of certain materials produces large amounts of trace contaminants. Since most of man's sub-systems carried aboard will not operate at 100% efficiency, incomplete oxidation may present a significant problem for the astronauts. Certain oxidative processes are hazardous, however, while similar ones are benign. For example, with methyl alcohol, only partial oxidation is achieved with the resultant formation of formaldehyde, whereas with ethyl alcohol, sealed-cabin environmental studies[2] indicate that its oxidation is complete, and there is no formation of toxic aldehydes.

Thermal degradation presents many inherent difficulties. In some instances, toxicants may be degraded to nontoxic materials, whereas the opposite is also true; probably the latter occurs more frequently. An example in a sealed atmosphere has been shown in the studies aboard nuclear submarines. Freon (CCl_2F_2) has been employed for air conditioning. At slightly elevated temperatures this material was decomposed into hydrogen fluoride, hydrogen chloride, chlorine gas, and fluorine. This is an example of a material which is generally considered to be nontoxic but produces several lethal by-products. At high temperatures Freon is broken down into carbonyl chlorides, known as phosgene in chemical warfare. Such degradation in the submarine has forced surfacing; however, in our space vehicle, one cannot anticipate having the privilege of calling off a mission when the environment becomes adverse.

Table 1 indicates the numerous variables which may affect an orbiting laboratory. Almost none of these has been clearly defined in our present ground investigations, as such endeavors must be undertaken in both locales to establish the role each will have in a manned space environment.

The interactions of all of the contaminants can show combined synergism and/or antagonism with all of the described factors. This would constitute a profile of combined biophysical and biochemical stressors. Certainly, atmospheric specialists are not overly familiar with any of the reactions just expressed for an orbiting station and the summation of such reactions would be only problematical. However, it must be assumed that the effects would be deleterious and would endanger man's performance and his space vehicle to the extent that they might be noxious, obnoxious, or even lethal with the possible consequence of aborting the mission.

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Weight Minimization of a Step Rocket by the Discrete Maximum Principle

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Nomenclature

c^n = propellant exhaust velocity of stage n
M^n = ratio of initial thrust to weight of rocket n
n = number of stages (appear as superscripts)
V = design velocity of rocket system
W_L = rocket system payload weight
w^n = initial gross weight of step n
d^n = portion of stage n jettison weight which is constant
$\alpha^n w^n$ = portion of stage n jettison weight dependent of step weight
$\beta^n M^n x^n$ = portion of stage n jettison weight dependent of thrust
$\sigma^n w^n$ = total jettison weight of stage n
x_1^n = initial gross weight of rocket $n = x_1^{n-1} + w^n$
θ^n = velocity ideally added during stage n
a^n = $-\beta^n M^n - \alpha^n$
b^n = $1 - \alpha^n$

Introduction

A SKETCH of a three-stage step rocket system is shown in Fig. 1. The difference of weight between two consecutive stages is called the step weight, and weight remaining after the jettison of the last stage is called the rocket payload. The purpose of such a rocket system is to attain the specified final velocity carrying the desired rocket payload. If the number of stages and the material and propellant for constructing the rocket system have been decided, the problem is to determine the optimum size of each stage, so that the gross weight of the system is minimized.

Two-stage[1] and three-stage[2] optimization problems have been solved by differential calculus; in order to make hand calculation possible, simplifying assumptions have been made which are quite reasonable for many problems, and the results are found to compare favorably with actual practice.[3] The simplifying assumptions, however, must be verified by comparison with solutions obtained by removing all or some of the assumptions. Also, many practical problems may have to be solved by use of other approaches, e.g., the dynamic programming algorithm.[3]

In the present work, the discrete maximum principle solution of the problem formulated by Dyke[3] is obtained, using his definitions of terms.

Formulation

When the discrete maximum principle algorithm is employed, the problem can be reformulated as follows (Fig. 2):

$$x_1^n/(x_1^{n-1} + \sigma^n w^n) = \exp(\theta^n/c^n) \quad (1)$$

If we assume that the jettison weight is a function h of w and θ and that c, M, d, α, β, and σ are constants for each step rocket,

$$\sigma^n w^n = h[w^n;\ \theta^n] \quad (2)$$

A combination of Eqs. (1) and (2) gives

$$x_1^n = T_1^n(x_1^{n-1};\ \theta^n) \quad (3)$$

For the first stage,

$$x_1^1 = T_1^1(w_L;\ \theta^N) \quad (4)$$

The optimization problem is to choose a sequence of θ^n at each step so that the initial gross weight of the rocket system is minimized and a designated velocity V for the rocket payload is attained.

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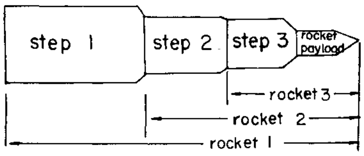

Fig. 1 A sketch of three-stage rocket system.

Defining a new variable

$$x_2^n = x_2^{n-1} - \theta^n = T_2^n(x_2^{n-1};\ \theta^n) \tag{5}$$

the problem is now to minimize x_1^N with the following boundary conditions: $x_1^0 = W_L$, $x_2^0 = V$, and $x_2^N = 0$. It can be shown that

$$\sum_{n=1}^{N} (x_2^{n-1} - x_2^n) = \sum_{n=1}^{N} \theta^n = x_2^0 - x_2^N = x_2^0 \tag{6}$$

Thus

$$\sum_{n=1}^{N} \theta^n = V$$

The essence of the discrete maximum principle[4] is to introduce a new set of variables defined by

$$z_i^{n-1} = \sum_{j=1}^{2} \frac{\partial T_j^n(x_k^{n-1};\ \theta^n)}{\partial x_i^{n-1}} z_j^n \tag{7}$$

$$i = 1, 2, \ldots, \qquad n = 1, 2, \ldots, N$$

where x_k^{n-1} is a shorthand writing for x_1^{n-1}, x_2^{n-1}. The Hamiltonian H^n is formed as follows:

$$H^n = \sum_{j=1}^{2} z_j^n T_j^n(x_k^{n-1};\ \theta^n) \tag{8}$$

It can be shown that

$$x_i^n = \frac{\partial H^n}{\partial z_i^n} \qquad z_i^{n-1} = \frac{\partial H^n}{\partial x_i^{n-1}} \tag{9}$$

Then the optimization problem becomes that of finding a sequence of $\{\theta^n\}$ to satisfy the following condition[4]:

$$H^n = \sum_{j=1}^{2} z_j^n T_j^n(x_k^{n-1};\ \theta^n) = \text{minimum} \tag{10}$$

where $0 \leq \theta^n \leq V$ with the boundary condition $z_1^N = 1$.

Computational Procedure

The general computational procedure usually takes less time and computer memories than those required by the dynamic programming calculation. Depending on the form of the performance or transformation function T_1^n, a simplification can often be made in the general working scheme. Consider the case[3] in which h has the functional form

$$\sigma^n w^n = \alpha^n w^n + \beta^n M^n x_1^n + d^n \tag{11}$$

i.e., for the stage n, total jettison weight is the sum of the jettison weight which depends on the step weight and that which depends on step thrust plus a constant term. For most cases this assumption is sufficiently valid for practical purposes.

Combining Eqs. (1) and (11) gives

$$x_1^n = (b^n x_1^{n-1} + d^n)/[\exp(-\theta^n/c^n) + a^n] \tag{12}$$

and x_2^n is given by Eq. (5).

To minimize x_1^N, Eqs. (7–10) are employed together with the boundary conditions given after Eq. (5). Thus

$$z_1^{n-1} = \frac{b^n z_1^n}{\exp(-\theta^n/c^n) + a^n} \tag{13}$$

$$z_2^{n-1} = z_2^n = z_2 \qquad z_1^N = 1 \tag{14}$$

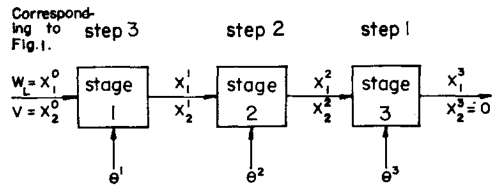

Fig. 2 Representation of stages for the maximum principle algorithm.

Hence,

$$H^n = z_1^n \left\{\frac{b^n x_1^{n-1} + d^n}{\exp(-\theta^n/c^n) + a^n}\right\} + z_2(x_2^{n-1} - \theta^n)$$

$$n = 1, 2, \ldots, N \tag{15}$$

The optimal sequence of $\{\theta^n\}$ occurs at the stationary points that are $\partial H^n/\partial\theta^n = 0$, or

$$\frac{\partial H^n}{\partial \theta^n} = -z_2^n + \frac{z_1^n}{c^n}\frac{(b^n x^{n-1} + d^n)\exp(-\theta^n/c^n)}{[\exp(-\theta^n/c^n) + a^n]^2} = 0$$

Solving for z_1^n,

$$z_1^n = \frac{z_2 c^n[\exp(-\theta^n/c^n) + a^n]^2}{(b^n x_1^{n-1} + d^n)\exp(-\theta^n/c^n)} \tag{16}$$

z_1^{n-1} is obtained similarly, with all superscripts reduced by 1. Inserting these into Eq. (13) gives

$$\frac{c^{n-1}[\exp(-\theta^{n-1}/c^{n-1}) + a^{n-1}]^2}{(b^{n-1}x_1^{n-2} + d^{n-1})\exp(-\theta^{n-1}/c^{n-1})} = \frac{b^n c^n[\exp(-\theta^n/c^n) + a^n]}{(b^n x_1^{n-1} + d^n)\exp(-\theta^n/c^n)} \tag{17}$$

From Eq. (16),

$$\exp(-\theta^n/c^n) + (b^n x_1^{n-1} - a^n x_1^n + d^n)/(x_1^n) \tag{18}$$

$$\exp(-\theta^n/c^n) + a^n = (b^n x_1^{n-1} + d^n)/(x_1^n) \tag{19}$$

and likewise for the previous stage, reducing all superscripts by 1.

Substituting these into Eq. (17) gives

$$x_1^n = \frac{1}{a^n}\left\{b^n x_1^{n-1} + d^n - \frac{b^n c^n x_1^{n-1}(b^{n-1}x_1^{n-2} - a^{n-1}x_1^{n-1} + d^{n-1})}{c^{n-1}(b^{n-1}x_1^{n-2} + d^{n-1})}\right\} \tag{20}$$

For the Nth stage, from Eqs. (13) and (14),

$$z_1^{N-1} = b^N/[\exp(-\theta^N/c^N) + a^N] \tag{21}$$

Substituting Eq. (14) into Eq. (16) gives

$$\frac{\exp(-\theta^N/c^N)}{[\exp(-\theta^N/c^N) + a^N]^2} = \frac{z_2 c^N}{(b^N x_1^{N-1} + d^N)} \tag{22}$$

For the $(N-1)$th stage, the same result is obtained, reducing all superscripts by 1.

Dividing Eq. (22) by the similar equation for the $(N-1)$th stage and inserting Eq. (21) for z_1^{N-1},

$$\frac{\exp(-\theta^N/c^n)}{\exp(-\theta^N/c^N) + a^N} = \frac{b^N c^N(b^{N-1}x_1^{N-2} + d^{N-1})\exp(-\theta^{N-1}/c^{N-1})}{[\exp(-\theta^{N-1}/c^{N-1}) + a^{N-1}]^2(b^N x_1^{N-1} + d^N)c^{N-1}} \tag{23}$$

For this particular example, the general computational procedure may be simplified as follows:

1) Since x_1^0 and x_2^0 are known, x_1^1 and x_2^1 can be evaluated from Eqs. (16) and (7) by assuming a value for θ^1.

2) Next, x^n can be calculated directly from Eq. (27) for stages 2 to $(N - 1)$. Then θ^n and x_2^n are calculated from Eqs. (16) and (7), respectively.

3) Finally, θ^N is obtained from Eq. (31) for the last stage (Nth stage). If x_2^N, calculated by substituting the computed θ^N into Eq. (7), is not zero, the procedure is repeated from the first step by assuming a new value for θ^1.

4) If the x_2^N thus obtained is zero, this sequence of θ^n is the optimal control actions sought. x_1^N can be secured from Eq.(16).

A similar working scheme may be followed by assuming a value of x_1^1 instead of θ^1.

Concluding Remarks

The maximum principle approach has the same advantage as the dynamic programming method in that it allows us to optimize rockets with three or more stages and to work with fewer assumptions than the conventional calculus method. Since there is no interpolation involved in the maximum principle solution, this method yields answers more accurate than those of the dynamic programming approach. The discrete maximum principle approach also requires much less computer memory capacity than the dynamic programming method because the solutions are not imbedded as in the case of the dynamic programming method. Usually the maximum principle algorithm requires less computational effort than dynamic programming algorithm.

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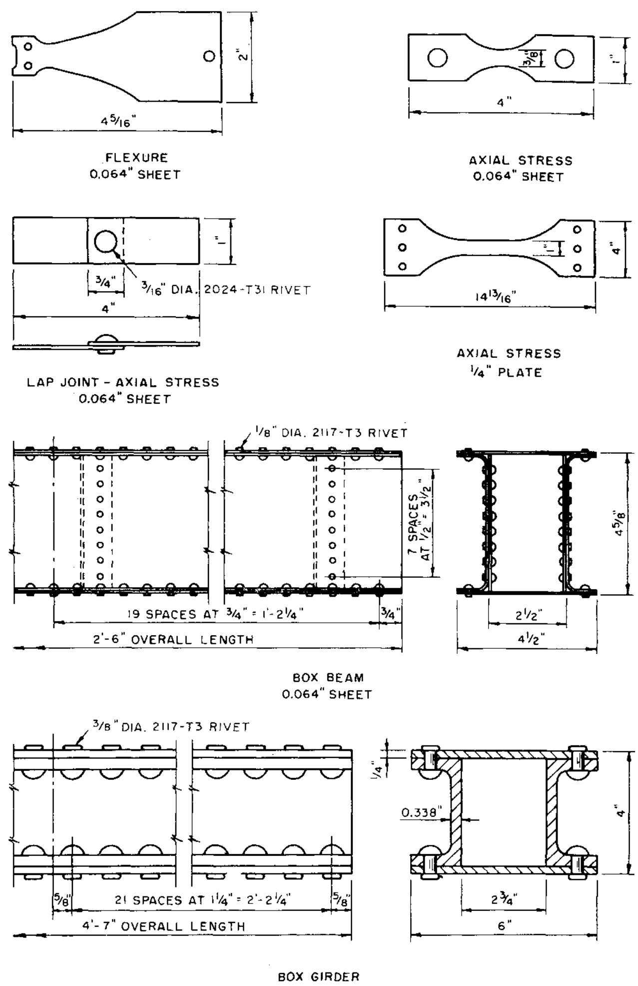

Fig. 1 Fatigue specimens.

Fatigue of Aluminum with Alclad or Sprayed Coatings

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Introduction

For improved resistance to corrosion, aluminum products of the higher strength alloys are often produced as alclad material, the surface layers of which consist of an alloy having greater resistance to corrosion than the core. Because such cladding is limited to sheet, plate wire, and tube, some use has been made of a metallized aluminum coating sprayed on the surface of more complicated shapes. Flexural fatigue strengths obtained at Alcoa Research Laboratories (Table 3.3.1(c) of Mil HDBK-5[1]) show that alclad sheet has substantially lower fatigue strength than bare sheet. Some rotating-beam fatigue tests showed that sprayed aluminum coatings also reduced the fatigue strength.

This paper shows that differences in fatigue strengths of bare and alclad plain sheet specimens are not present to the same degree in built-up construction of these materials; i.e., the fabrication stress raisers overshadow the effect of the coating on fatigue strength.

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Specimens and Test Procedures

Table 1 gives the essential data for the materials used. The details of the specimens are shown in Fig. 1. The material to be spray coated was prepared by blasting it with angular steel grit. Then a 99.5% aluminum coating was applied by an atomizing spray unit. The plate and channels for the box girders were spray coated before the riveting was accomplished.

The 0.064-in.-thick axial stress specimens were tested in 5-kip capacity Krouse direct-stress fatigue testing machines at stress ratios (R = minimum stress/maximum stress) of -1.0, -0.5, 0, 0.5, 0.75, and 0.90. The zero-stress ratio, axial-stress tests of the 0.25-in. plate were conducted in a similar 15-kip capacity machine. The sheet flexure tests were performed in Sontag SF-2 constant load flexural fatigue machines at a -1.0 stress ratio. The -1.0 stress ratio, axial-load fatigue tests of riveted lap joints were performed using special adaptors in ARL rotating-beam fatigue machines or in a 5-kip Krouse machine. The tests of lap joints and box beams were described in Refs. 2–4. The box girders and box beams were subjected to flexural tests in Templin structural fatigue machines. Most of the box beams were loaded through a 4-in.-long block located centrally on a 25-in. span. The loads were applied to the girders through fixtures attached to the webs of the channels at points 4-in. from midspan of a 40-in. span. For the box girders, R was 0, and for the box beams, R was -1.0.

The loads applied by the various fatigue machines were maintained by periodic checks. The tests were considered complete when the automatic cutoff switch on the machine stopped the test and a visible crack was observed.